Week 7 Thursday Worksheet - Graphing

Instructions. Follow the instructions given by your TA. You are not expected to finish all the problems. :)

1. **Steps of Graphing:**

   (0. if there is a square root or denominator $\Rightarrow$ find the domain)

   (a) x intercept $y=0,\ x=\cdots$

   y intercept $x=0,\ y=\cdots$ (if any)

   (b) Asymptotes:

   Vertical Asymptotes: answer looks like $x=\cdots$

   Horizontal Asymptotes: $\lim_{x\to\pm\infty} f(x)$. answer looks like $y=\cdots$.

   (c) $f'(x) = 0$ critical points ________

   Intervals where $f'(x) > 0$, $f'(x) < 0$.

   $\begin{cases} f'(x) > 0 & \nearrow \\ f'(x) < 0 & \searrow \end{cases}$

   (d) $f''(x)$ changes sign inflection points ________

   Intervals where $f''(x) > 0$, $f''(x) < 0$.

   $\begin{cases} f''(x) > 0 & \text{concave up} \\ f''(x) < 0 & \text{concave down} \end{cases}$

   (e) Draw the table

   (f) Draw the graph (according to the table)

2. Draw the graph of $f(x) = x^3 - 3x$

   (a) x intercept $x^2(x-3)=0 \Rightarrow x=0,3$

   y intercept $x=0 \Rightarrow y=0$ (if any)

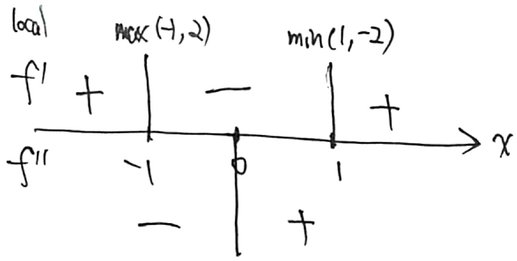
   (b) Asymptotes? None

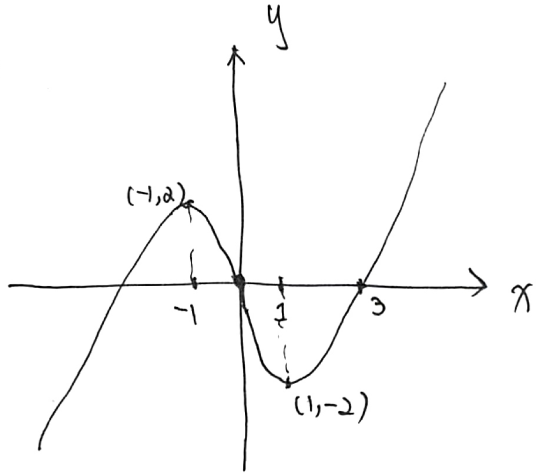
   (c) $f'(x) = 3x^2-3$ critical points $x=\pm 1$

   (d) $f''(x) = 6x$ inflection points $x=0$

   (e) Draw the table

   (f) Draw the graph (according to the table)





3. Draw the graph of $f(x) = x + \frac{1}{x}$ (Domain $x \neq 0$)

① x-int : none

y-int : none

② VA : $x=0$

HA : None.

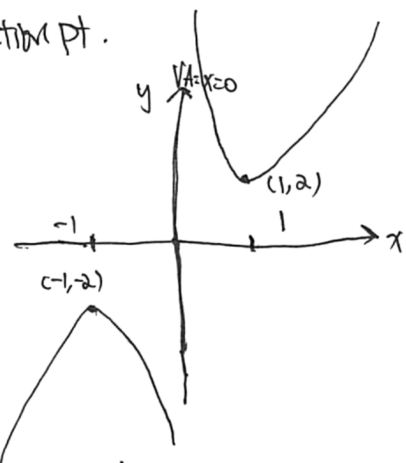
③ $f'(x) = 1 - \frac{1}{x^2}$

$f'(x)=0 \quad 1 = \frac{1}{x^2}$, $x = \pm 1$ critical pts

④ $f''(x) = 2x^{-3} = \frac{2}{x^3}$ $x=0$ inflection pt.

⑤ local max $(-1,-2)$ min $(1,2)$

f': $+$ | $-$ | $+$; f'': $-$ | $+$; marks at -1, 0, $+1$ on the x axis



4. Draw the graph of $f(x) = \frac{x^2}{x^2-1}$ Domain: $x \neq \pm 1$

① x int : $x=0$

y int : $x=0, y=0$

② Asym.

VA : $\boxed{x = \pm 1}$

HA : $\lim_{x \to \pm\infty} \frac{x^2}{x^2-1} = 1$ $\boxed{y=1}$

Check: $\lim_{x \to +1^+} f(x) = +\infty$, $\lim_{x \to +1^-} f(x) = -\infty$

$\lim_{x \to -1^+} f(x) = -\infty$, $\lim_{x \to -1^-} f(x) = +\infty$.

③ $f'(x) = \frac{2x(x^2-1) - 2x \cdot x^2}{(x^2-1)^2}$

$= \frac{-2x}{(x^2-1)^2}$

$f'(x)=0$ critical pts: $x=0$

④ $f''(x) = \frac{-2(x^2-1)^2 - 2(x^2-1) \cdot 2x(-2x)}{(x^2-1)^4}$

$= \frac{-2(x^2-1) + 8x^2}{(x^2-1)^3} = \frac{6x^2+2}{(x^2-1)^3}$ inflection pt $x = \pm 1$

⑤ local max $(0,0)$

f': $+$ | $-$; f'': $+$ | $-$ 0 | $+$